Calculus (Type B)

Part I：Choice. Select ONLY ONE answer choice from a list of four choices.

(5 points for each problem)

1. Find the value of $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^4\frac{1}{n}$

   (A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{6}$.

   Ans: (C)

2. Find $\int \sin^{10}x\cos x\,dx$.

   (A) $\frac{1}{11}\sin^{11}x+C$ (B) $\frac{-1}{11}\sin^{11}x+C$ (C) $\frac{1}{11}\cos^{11}x+C$ (D) $\frac{-1}{11}\cos^{11}x+C$.

   Ans: (A)

3. Assume $f'(x)=8x^3-12x^2-3$ and $f(-1)=10$. Find $f(1)$.

   (A) -10 (B) -4 (C) 2 (D) 5

   Ans: (B)

4. Find $\int_1^e \frac{\ln\sqrt{x}}{x}dx$

   (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 2 (D) 4

   Ans: (A)

5. Find $\int_0^{2\pi} x^2\sin x\,dx$

   (A) $-\frac{8}{3}\pi^3$ (B) $-\frac{4}{3}\pi^3$ (C) $-2\pi^2$ (D) $-4\pi^2$

   Ans: (D)

6. Find $\int_0^{\infty} e^{-x}\cos x dx$.

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

Ans: (A)

7. $\int_{-1}^{1}\frac{\tan x}{1+x^2+x^4}dx$ equals

(A) 2 (B) 0 (C) -3 (D $-\frac{3}{4}$

Ans: (B)

8. Find $\int_0^1 \frac{x}{1+x^2}dx$=? (A) $\frac{\pi}{4}$ (B) $\ln 2+\frac{1}{2}$ (C) $\ln\sqrt{2}$ (D) $\frac{\pi}{4}+\ln\frac{\pi}{2}$

Ans: (C)

9. Using integration by parts find $\int_1^e x\ln x dx$=?

(A) $\frac{1}{4}\left(e^2+1\right)$(B) e^2+1 (C) e^2-11 (D) $\frac{1}{4}\left(e^2-1\right)$

Ans: (A)

10. Find the area of the region between the parabola $y=x^2$ and the line $y=x$

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$。

Ans: (C)

11. Find the volume of the solid generated by revolving the region bounded by the parabolas $y=x^2$ ，and the line $x=2$ about the x-axis.

(A) $\frac{22\pi}{5}$ (B) $\frac{32\pi}{5}$ (C) $\frac{42\pi}{5}$ (D) 6π

Ans: (B)

12. Find $\int_{\ln 2}^{\ln 5} \frac{e^{2x}}{e^{2x}-1}dx$?

(A) $\frac{1}{2}\ln 8$ (B) $\frac{1}{2}\ln 12$ (C) $\ln 4$ (D) $\ln 8$

Ans: (A)

Part II：Fill in the blanks

a. (6pts.) Find $\int_0^{2\pi}(\sin x+\cos x)^2 dx =$ ①π .

Ans: 2π

b. (7pts.) Assume $f(x)=\int_2^{x^2}\sqrt{9-t^2}\,dt$. Find $f'(2)$. ②③ .

Ans: $4\sqrt{7}$

c. (7pts.) If $f(0)=5, f(3)=1, f'(0)=1,$ and $f'(3)=-2$, find $\int_0^3 xf''(x)\,dx =$ ④⑤ .

Ans: -2

d. (7pts.) Find the area of the region bounded by $y=(x-4)(x+2)$, $y=0$ between $x=0$ and $x=3$.

⑥⑦ .

Ans: 24

e. (6pts.) $\int_0^{\infty} xe^{-x}dx =$ ⑧ .

Ans:1

f. (7pts.) Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$, $y=2$ and $x=0$ (or y-axis) about the y-axis. $\frac{⑨⑩\pi}{5}$

Ans：$\frac{32\pi}{5}$